Answer Key

Exponential Functions Unit Review

1. Records at the Universal Video store show that sales of new DVDS are greatest in the first month after the release date. In the second month, sales are usually only about one-third of sales in the first month. Sales in the third month are usually only about one-third of sales in the second month, and so on.

   a. If Universal Video sells 31886460 copies of one particular DVD in the first month after its release, how many copies are likely to be sold in the second month? In the third month? Use the table below to help you answer the questions.

Number of Months	0	1	2	3	4	5
Number of DVD Sales	31886460	10628820	3542940	1180980	393660	131220

   b. What NOW-NEXT and "y = " rules predict the sales in the following months?

   $y = 31886460\,(1/3)^x$ Next = Now · 1/3 Start @ 31886460

   c. Use your equations to predict how many DVDs are in the 12th month?

   60 DVD'S

   d. In what month are sales likely to first be fewer than 5 copies?

   between 14 - 15 months.

   e. How would your answers to parts a – d change for a different DVD that has first month sales of 26572050 copies?

Number of Months	0	1	2	3	4	5
Number of DVD Sales	26572050					

   NOW-NEXT rule: Next = Now · 1/3 Start @ 26572050

   Y = $26572050\,(1/3)^x$

   Number of DVDs in the 12th month: 50 DVD'S

   First month in which fewer than 5 copies are sold: between 14-15

2. Find the next three terms in each sequence. Identify each as arithmetic, geometric, or neither. For each arithmetic or geometric sequence, find the common difference or common ratio. Then write a NOW-NEXT rule to describe the sequence.

   a. 14, 11, 8, 5, 2 . . . arithmetic diff = -3 Start 14 Next = Now - 3 -1, -4, -7

   b. 3,000, 300, 30, 3 . . . geometric ratio = 1/10 Start 3000 Next = Now · $\frac{1}{10}$ $\frac{3}{10}$, $\frac{3}{100}$, $\frac{3}{1000}$

   c. 5, 6, 8, 11, 15, 20 . . . neither

3. Tell whether each situation produces an arithmetic sequence, a geometric sequence, or neither.

   a. The temperature rises at the rate of 0.75°F per hour. arithmetic

   b. A person loses 2 lbs each month. Arithmetic

   c. A toadstool doubles in size each week. geometric

   d. A person receives a 6% raise each year. geometric

~~4.~~ State the y-intercept. (HINT: think about what x needs to be)

a. $Y = 2^x - 3$ b. $y = 3^{x+1}$ c. $y = (1/3)^x$

(0, -2) (0, 3) (0, 1)

5. You may have heard of athletes being disqualified from competitions because they have used anabolic steroid drugs to increase their weight and strength. These drugs are dangerous and leave the body slowly. With an injection of the steroid cyprionate, about 90% of the drug and its by-products will remain after a ~~second~~ day, and so on. Suppose that an athlete tries steroids and injects a dose of 100 mg of cyprionate.

a. Make a table showing the amount of the drug remaining at various times.

Number of Days	0	1	2	3	4	5
Amount of Cyprionate	100	90	81	72.9	65.61	59.049

b. Make a plot of the data in part a on your graph paper and write a short description of the pattern shown. Curves downward

c. Write two rules that describe the amount of steroid in the blood.

NOW-NEXT rule: next = now · .9 Start @ 100

Y = $100(.9)^x$

d. Use one of the rules in part c to estimate the amount of steroid left after 0.5 days and 8.5 days.

.5 → 94.9

8.5 → 40.84

e. Estimate, to the nearest tenth of a day, the half-life of cyprionate. between 6.5-6.6

≈ 6.6 days.

f. How long will it take the steroid to be reduced to only 1% (1 mg) of its original level in the body? between 43-44 days.

6. For each of the following rules, decide whether the function represented is an example of: an increasing linear function, a decreasing linear function, an exponential growth function, an exponential decay function, or neither a linear or exponential function.

a. $Y = 5(0.4^x)$

decreasing exponential

b. NEXT = 5 • NOW

increasing exponential

c. $Y = 5 - 0.4x$

decreasing linear

d. NEXT = NOW – 5

decreasing linear

e. $Y = 5/x$

neither.

f. NEXT = 0.4 • NOW

decreasing exponential.

7. In 2000, the number of people worldwide living with HIV/AIDS was estimated at more than 36 million. That number was growing at an annual rate of about 15%.

   a. Make a table showing the projected number of people around the world living with HIV/AIDS in each of the ten years after 2000, assuming the growth rate remains 15% per year.

Years after 2000	0	1	2	3	4	5	6	7	8	9	10
AIDS Cases (in millions)	36	41.4	47.6	54.8	63	72.4	83.3	95.8	110.1	126.6	145.6

   b. Write two different kinds of rules that could be used to estimate the number of people living with HIV/AIDS at any time in the future.

   NEXT = now $\cdot$ 1.15   start @ 36

   Y = $36(1.15)^x$

   c. Use the rules from part b to estimate the number of people living with HIV/AIDS in 2015.

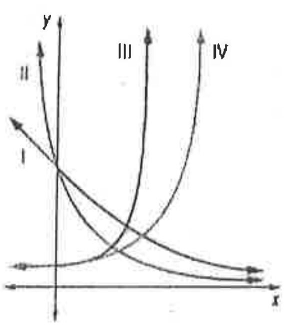
   292.93 million people.

   d. What factors might make the estimate of part c an inaccurate forecast?

   answers vary.

8. The graphs, tables, and rules below model four exponential growth and decay situations. For each graph, there is a matching table and a matching rule. Use what you know about the patterns of exponential relations to match each graph with its corresponding table and rule. In each case, explain the clues that can be used to match the items without any use of a graphing calculator or computer.

Graphs



Tables

A

x	1	2	3	4
y	40	16	6.4	2.56

~~B~~

x	1	2	3	4
y	30	90	270	810

C

x	1	2	3	4
y	60	36	21.6	12.96

~~D~~

x	1	2	3	4
y	20	40	80	160

Rules

(1) $y = 100(0.6^x)$

(2) $y = 100(0.4^x)$

~~(3) $y = 10(2^x)$~~

~~(4) $y = 10(3^x)$~~

I, C, 1
II, A, 2
III, B, 4
IV, D, 3

9. Exponential functions can be expressed by a rule relating x and y values and by a rule relating NOW and NEXT values.

   a. Write a general rule for an exponential function y = $a \cdot b^x$

   b. Write a general rule relating NOW and NEXT for an exponential function:

      Next = now · b start at a

   c. What do the parts of the rules tell you about the problem situation?

      a - Starting value   b - common ratio.

   d. How do you decide whether a given exponential function rule will describe growth or decay, and why does your decision rule make sense?

      $0<b<1$ is decay & $b>1$ is growth

10. You plan to deposit $100 into a savings account. The three options offered by your bank are described below. Determine how much money you would have in each option after 12 years. Round your answer to the nearest cent.

   a. Option 1: 4% interest compounded annually $160.10

      $y = 100(1.04)^{12}$

   b. Option 2: 3.9% interest compounded semiannually $158.96

      $y = 100(1.0195)^{24}$

   c. Option 3: 3.8% interest compounded monthly $157.66

      $y = 100\left(6019/6000\right)^{144}$

   d. Which option will you choose? Explain your answer.

      option 1. More money is made after the 12 years with this plan.

11. The half life of Cs-137 is 30.2 years. If the initial mass of the sample is 1.00kg, how much will remain after 151 years?

    $y = 1\left(\frac{1}{2}\right)^5 = .03125$ kg.

12. A 0.5g sample of radioactive Iodine-131 has a half life of 8.0 days. After 40 days, how much is left?

    $y = .5\left(\frac{1}{2}\right)^5 = .015625$ g

Adapted from *Core-Plus Mathematics*, Glencoe McGraw-Hill: Columbus, OH, 2008.

1.038
1.00316 = 6019/6000

Name: ______________________ Date: ______________ Period: __________

Unit 5 Review - Exponential Functions

Identify if the sequence is arithmetic or geometric. Identify if there is a common ratio or difference and what it is. Next, write a NOW-NEXT rule. Then find the next three numbers in each sequence.

1. $48, 12, 3, \frac{3}{4}, \frac{3}{16}, \ldots$

Arithmetic OR Geometric

Common Ratio = 1/4

Now-Next Rule start = 48 Next = now · 1/4

Next 3 terms 3/64 3/256 3/1024

2. 8.7, 5.2, 1.7, -1.8, ...

Arithmetic OR Geometric

Common difference = -3.5

Now-Next Rule Start = 8.7 Next = Now - 3.5

Next 3 terms -5.3, -8.8, -12.3

3. $y = 4 \cdot \left(\frac{1}{2}\right)^x$

a. Identify the y-intercept (0,4)

b. As the x-value increases by 1, the y-value multiples by 1/2.

For each exponential function below explain how it is translated from the function $y = 2^x$.

	Equation	How is it translated from the original function $y = 2^x$?
~~4.~~	$y = 2^x + 1$	
~~5.~~	$y = 2^{x+1}$	
~~6.~~	$y = 2^{x-1} - 1$	

7. Suppose a population of 300 beetles doubles in size every 6 months. How many beetles will there be after 3 years?

a. Write an equation to model the beetle population growth.

Let x represent number of 6 month periods   # of years

Let y represent number of beetles   ↓

Equation: $y = 300 \cdot (2)^x$ $y = 300 \cdot 2^{2x}$

b. Predict the population after 3 years. 19,200 beetles.

8. A truck costs $23,500 and decreases in value by 15% per year.

a. Write an equation to model the value of the truck.

Let x represent number of years.

Let y represent value of truck

Equation: $y = 23500(.85)^x$

b. Predict the value after 4 years. $12,267.15

9. $2,700 is put in an account and earns 10% interest annually. Find the balance in the account.

a. Write an equation to model the balance when compounded annually.

Let x represent # of years

Let y represent balance in account.

Equation: $y = 2700(1.10)^x$

b. Predict the balance after 8 years. $5787.69

c. Write an equation to model the balance when compounded semi-annually.

Let x represent # of 6 month periods   # of years.

Let y represent balance in account   ↓

Equation: $y = 2700(1.05)^x$ $y = 2700(1.05)^{2x}$

d. Predict the balance after 8 years. $5893.76

e. Write an equation to model the balance when compounded quarterly.

Let x represent # of 3 month periods

#of years

↓

$y=2700(1.025)^{4x}$

Let y represent account balance

Equation: $y=2700(1.025)^x$

f. Predict the balance after 8 years. $5950.14

10. Suppose a laboratory has a 32 g sample of polonium-210. The half-life of polonium-210 is about 30 days.

a. How many half-lives of polonium-210 occur in 90 days? 3

b. How much polonium is in the sample after 90 days? 4 grams.

$y=32(1/2)^3$

Given the table determine if the relationship is linear or exponential and write appropriate equation. Round to the nearest hundredth, if necessary.

11.

X	Y
-2	22400
-1	5600
1	350
3	21.875

Model: Linear Exponential

Equation: $y=1400(1/4)^x$

12.

X	Y
-4	2
-2	2.5
4	4
10	5.5

Model: Linear Exponential

Equation: $y=1/4x+3$